Roll No. .... **Total Pages : 04**

BT-I/D-21 **41046**

CALCULUS & LINEAR ALGEBRA

BS-133-A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Prove that :

$$\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$$

where β represents the Beta function and Γ is the gamma function.

(b) Find the volume formed by the revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$, about the x-axis.

2. (a) Show that :

$$\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x} = -\frac{e}{2}$$

(b) State and prove Rolle's theorem.

Unit II

3. (a) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$, and I is identity matrix of order 3, evaluate $A^2 - 3A + 9I$.

(b) Solve the following system of equations using Cramer's rule :

$$x + y + z = 4$$
$$x - y + z = 0$$
$$2x + y + z = 5$$

4. (a) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

(b) If $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, prove that :

$(AB)^{-1} = B^{-1}A^{-1}$.

Unit III

5. (a) Show that the vectors $v_1 = (2, -1, 0)$, $v_2 = (1, 2, 1)$ and $v_3 = (0, 2, -1)$ are linear independent. Also express the vector (3, 2, 1) as a linear combination v_1, v_2, v_3.

   (b) For what value of k (if any) the vector $v = (1, -2, k)$ can be expressed as linear combination of vectors $v_1 = (3, 0, -2)$ and $v_2 = (2, -1, -5)$ in $R^3(R)$.

6. (a) Show that the transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (x, y)$ is a linear transformation.

   (b) If $T : U(F) \to V(F)$ is a linear transformation, then show that :

$$\dim(R(T)) + \dim(N(T)) = \dim U$$

Unit IV

7. (a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

   (b) In an inner product space, if $\|u + v\| = \|u\| + \|v\|$, then show that u, v are linear dependent.

8. (a) If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal, find a, b and c.

(b) Express the matrix $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$ as sum of a symmetric and skew symmetric matrix.